THE SIXTH FORM
AT WOODHOUSE GROVE

Maths/Maths and Further Maths

TASK 1 - KEY SKILLS

After three weeks of Year 12 maths you will sit a 'key skills test'. The main purpose of the test is to identify pupils who will require a greater level of support in order to be successful in A-Level maths. The test will cover the following topics, most of which should be familiar to you from IGCSE/GCSE:

- **algebraic manipulation** eg algebraic fractions, expanding, factorising…
- **solving linear equations and changing the subject of a formula**
- **solving quadratics by factorising, completing the square and using the quadratic formula**
- **laws of indices**
- **manipulating surds**
- **simultaneous equations** (two linear or one linear/one non-linear)

Corbett maths will help

It's important that you are confident with the topics so that you can access the new content at A-level maths. The following questions give you practice at these key skills. Each topic has a list of videos that will help you. These are located on www.corbettmaths.com or use the QR code shown. Answers are at the end of these pages.

Section A: algebraic manipulation
(video #14)
Expand the following brackets and simplify.

1) $5a - 4(3a - 1)$

2) $(x + 2)(x + 3)$

3) $(2x + 3y)(3x - 4y)$

4) $4(x - 2)(x + 3)$

5) $(2y - 1)(2y + 1)$

6) $(3 + 5x)(4 - x)$

7) $(7x - 2)^2$

Section B: algebraic manipulation
(videos #118, #119, #120)
Factorise the following

1) $2x^3 - 6x^2$

2) $5y(y - 1) + 3(y - 1)$

3) $x^2 - x - 6$

4) $x^2 + 6x - 16$

5) $2x^2 - 3x$

6) $7y^2 - 10y + 3$

7) $4x^2 - 25$

Section C: changing the subject *(videos #7, #8)* Make the letter in brackets the subject of each of these formulae:

1) $y = \frac{x+5}{4}$ (x) 2) $r = a + bt^2$ (t) 3) $\frac{x}{a} = 1 + \frac{x}{b}$ (x)

Section D: simultaneous equations *(videos #295, #296, #298)* Show all your working

1) $9x - 2y = 25$
$4x - 5y = 7$

2) $x + y = 3$
$x^2 + y^2 = 25$

Section E: solving linear equations *(videos #110-113)* Solve the following equations, showing each step in your working:

1) $2x + 5 = 19$

2) $5 - 7x = -9$

3) $11 + 3x = 8 - 2x$

4) $7x + 2 = 4x - 5$

5) $5(2x - 4) = 4$

6) $4(2 - x) = 3(x - 9)$

7) $\frac{1}{2}(x+3) = 5$

8) $\frac{2x}{3} - 10 = \frac{x}{5} + 4$

Section F: solving quadratic equations

By Factorising *(video #266)*

1) $x^2 + 3x + 2 = 0$

2) $4 - x^2 = 0$

3) $6 + 2x = 8x^2$

By Completing the Square *(video #267a)*
(leave answers as surds)

1) $x^2 + 6x + 4 = 0$

2) $x^2 - 7x - 1 = 0$

Using the Quadratic Formula *(video #267)*
(round answers to 3sf)

1) $x^2 + 7x + 9 = 0$

2) $3x^2 = 15x - 16$

Section G: indices *(video #172-175)* non-calculator. Work out the value of:

1) $27^{1/3}$

2) $\left(\frac{1}{9}\right)^{1/2}$

3) 5^{-2}

4) $8^{-2/3}$

Simplify each of the following: *(video #17)*

5) $2a^{1/2} \times 3a^{5/2}$

6) $(x^2 y^4)^{1/2}$

Section H: surds *(videos #305-308)* Without using a calculator…

Simplify

1) $\sqrt{72}$

2) $\sqrt{24} + \sqrt{54}$

Expand and simplify

3) $\sqrt{2}(3 + \sqrt{5})$

4) $(3 - \sqrt{5})(3 - 2\sqrt{5})$

5) $(8 - \sqrt{2})(8 + \sqrt{2})$

Rationalise the denominator of

6) $\frac{4}{\sqrt{8}}$

7) $\frac{2 + \sqrt{3}}{\sqrt{3} - 1}$

TASK 2 - Increase your depth of knowledge

Knowing how to do things like completing the square is an important skill to have but having a deeper understanding of how, why and when is much more useful

Below are several tasks to have a go at to help increase your depth of knowledge - and give you an advantage when you join us in September. You may need to play around with DESMOS which is free to download and worth getting to grips with.

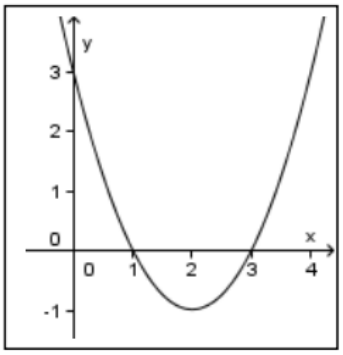
Completing the square: Think about all the quadratics curves with equation $y = (x + a)^2 + b$. Think also about these three properties.

A: The turning point has a positive x-value.

B: The turning point has a positive y-value.

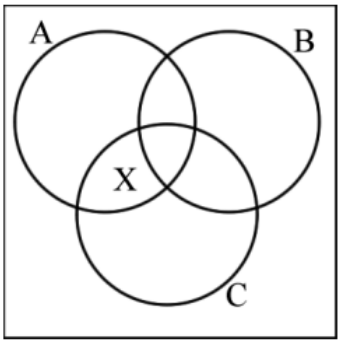
C: The y-intercept is positive.

The quadratic curve shown on the right satisfies properties A and C but not B. Its equation can be written as $y = (x - 2)^2 - 1$



You could write this in the region marked with an X in the Venn diagram below.

- ➔ Can you find a quadratic which doesn't satisfy any of the properties?
- ➔ Write this in the region outside the three circles.
- ➔ Can you find one equation for each of the other six regions?
- ➔ Is it possible to find an equation for every region?



Desmos will help here!

Using the digits 0 to 9 at most one time each, fill in the boxes to create three equations that produce the exact same parabola.

$$y = (x + \square)^2 - \square$$
$$y = (x + \square)(x + \square)$$
$$y = x^2 + \square x + \square$$

Using the digits 1 to 9 at most one time each, fill in the boxes to create a quadratic in factored form with the lowest minimum.

$$y = \square(x - \square)(x - \square)$$

Investigating solutions to trig equations (MEI Desmos tasks for AS maths page 9).
(https://mei.org.uk/files/ict/desmos-as-core-tasks.pdf#page=9)

1. Go to desmos.com
2. Press the graph setting wrench and set the *Range: -400<x<800, Step: 90, -2<y<2, Angles: degrees*
3. Add the graph:$y = \sin x$
4. Add the graph: $y = 0.8$
5. Click on the points of intersection to see their values

Questions for discussion

• What symmetries are there in the positions of the points of intersection?

• How can you use these symmetries to find the other solutions based on the value of $\sin^{-1} k$ given by your calculator? (This is known as the "principal value".)

Problem (Try the problem just using the $\sin^{-1}$ function on your calculator first then check it using the software)

Solve the equation: $\sin x = 0.2 \ (-360 \leq x \leq 720)$

Further Tasks

• Investigate the symmetries of the solutions to $\cos x = k$ and $\tan x = k$

• Investigate the symmetries of the solutions to $\sin 2x = k$

Check out this link for some history about trigonometry and the stars!
https://undergroundmathematics.org/trigonometry-triangles-to-functions/from-stars-to-waves

Further reading:

- ➔ https://www.gapminder.org/ - world stats as you have never seen them
- ➔ STEP UP from CIMT to help you prepare your key skills.http://www.cimt.org.uk/projects/mepres/step-up/index.htm
- ➔ NRich http://nrich.maths.org/secondary-upper
- ➔ Numberphile youtube channel https://www.youtube.com/user/numberphile
- ➔ Underground maths https://undergroundmathematics.org/
- ➔ Why do buses come in threes? – Rob Easterway
- ➔ Fermat's Last Theorem – Simon Singh
- ➔ Alex's Adventures in Numberland – Alex Bellos
- ➔ The Simpsons and their Mathematical Secrets – Simon Singh

Using the digits 1 to 9, at most one time each, fill in the boxes so that the two functions are equivalent

$$f(x) = \square x + \square$$

$$g(x) = \square x^2 + \square x + \square$$

$$f(\square) = g(\square)$$

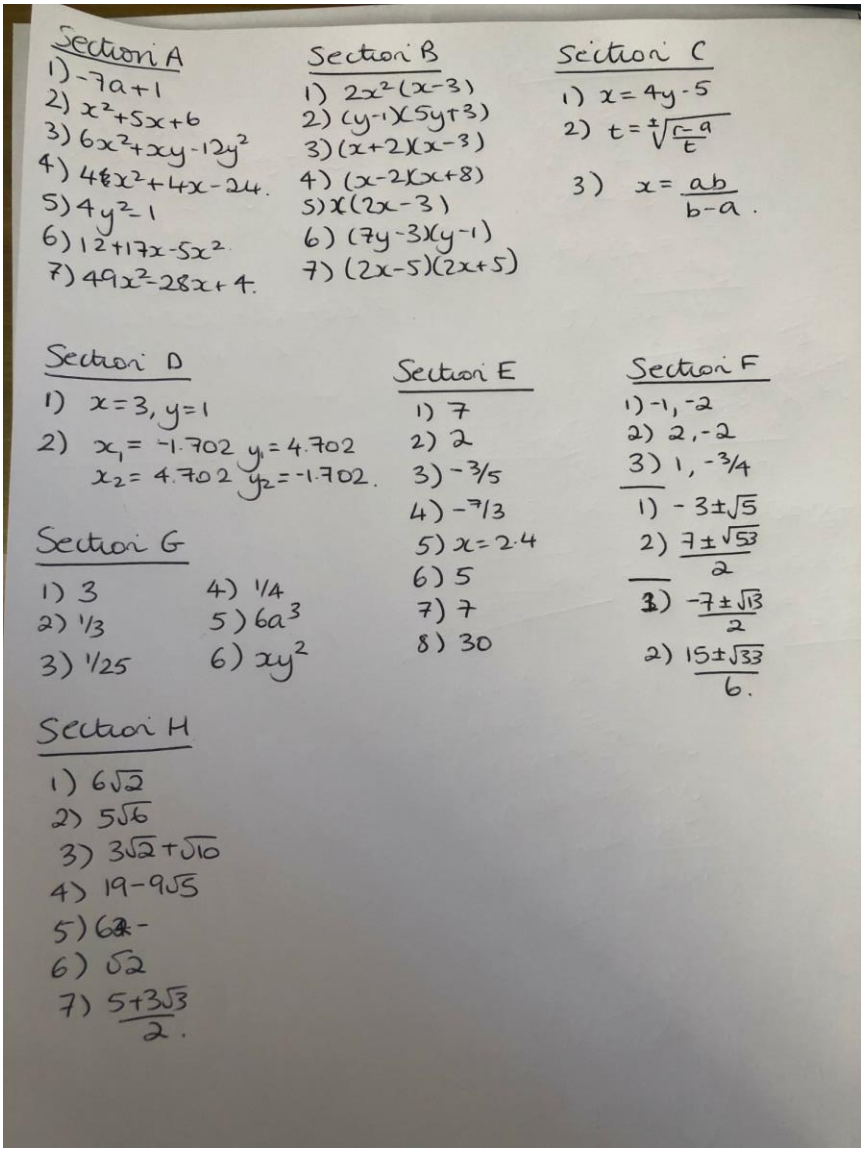
Using the digits 1 through 9, at most one time each, create a system of equations that has no solutions.

$$\square x + \square y = \square$$

$$\square x + \square y = \square$$

$$\square x + \square y = \square$$

Answers



Section A

1) $-7a+1$
2) x^2+5x+6
3) $6x^2+xy-12y^2$
4) $48x^2+4x-24$.
5) $4y^2-1$
6) $12+17x-5x^2$.
7) $49x^2-28x+4$.

Section B

1) $2x^2(x-3)$
2) $(y-1)(5y+3)$
3) $(x+2)(x-3)$
4) $(x-2)(x+8)$
5) $x(2x-3)$
6) $(7y-3)(y-1)$
7) $(2x-5)(2x+5)$

Section C

1) $x=4y-5$
2) $t=\pm\sqrt{\frac{r-9}{E}}$
3) $x=\frac{ab}{b-a}$.

Section D

1) $x=3, y=1$
2) $x_1=-1.702$ $y_1=4.702$
   $x_2=4.702$ $y_2=-1.702$.

Section E

1) 7
2) 2
3) $-3/5$
4) $-7/3$
5) $x=2.4$
6) 5
7) 7
8) 30

Section F

1) $-1, -2$
2) $2, -2$
3) $1, -3/4$

1) $-3\pm\sqrt{5}$
2) $\frac{7\pm\sqrt{53}}{2}$

1) $\frac{-7\pm\sqrt{13}}{2}$
2) $\frac{15\pm\sqrt{33}}{6}$.

Section G

1) 3
2) $1/3$
3) $1/25$
4) $1/4$
5) $6a^3$
6) xy^2

Section H

1) $6\sqrt{2}$
2) $5\sqrt{6}$
3) $3\sqrt{2}+\sqrt{10}$
4) $19-9\sqrt{5}$
5) $6\sqrt{2}-$
6) $\sqrt{2}$
7) $\frac{5+3\sqrt{3}}{2}$.